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澳門科技大學
UNIVERSIDADE DE CIÊNCIA E TECNOLOGIA DE MACAU
MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

澳門四高校聯合入學考試 (語言科及數學科)

Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2018 年試題及參考答案
2018 Examination Paper and Suggested Answer

數學正卷 Mathematics Standard Paper

<u>第一部分</u> 選擇題。請選出每題之*最佳*答案。

1. 若 $X=\{6^n-5n-1\,|\,n\in\mathbb{Z}^+\}$ 和 $Y=\{25n-25\,|\,n\in\mathbb{Z}^+\}$，則

A. $X\subset Y$　　B. $Y\subset X$　　C. $X=Y$　　D. $X\cap Y=\varnothing$　　E. 以上皆非

2. 將 $2x^3+x^2-29x+40$ 除以 $2x-5$，則餘數為

A. -5　　B. 5　　C. -20　　D. 20　　E. 30

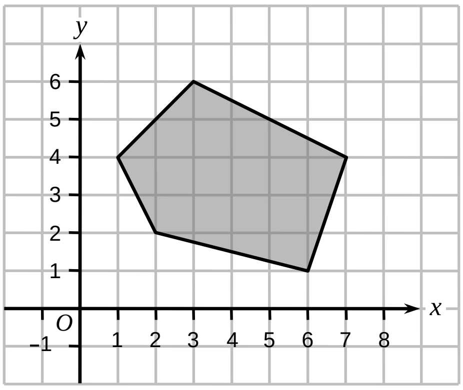
3. 若方程 $3x^2-4x+k=0$ 的兩根之差是 $\frac{5}{3}$，則 $k=$

A. $\frac{2}{3}$　　B. 3　　C. $-\frac{1}{6}$　　D. $-\frac{3}{4}$　　E. 以上皆非

4. $\dfrac{a^3b^{-2}c^2}{\left(2a^{-1}b^2c\right)^3}=$

A. $\dfrac{1}{8b^4c}$　　B. $\dfrac{a}{8b^4c}$　　C. $\dfrac{a^6}{8b^8c}$　　D. $\dfrac{a^4}{8b^4c}$　　E. $\dfrac{a^6b^3}{8c}$

5. 若 (x, y) 為下圖中陰影區域 (包括邊界) 任何一點，則 $M=3x+2y-6$ 的最大值為



A. 4　　B. 14　　C. 23　　D. 32　　E. 以上皆非

6. 若 $a, b>1$，則 $\log_a\left(\dfrac{a}{b}\right)+\log_b\left(\dfrac{b}{a}\right)$ 的最大值為

A. -2　　B. 0　　C. 2　　D. 3　　E. 4

7. 若 $\dfrac{x}{y-x}=\dfrac{1+y}{y}$，則 $x=$

A. $y+y^2$　　B. $\dfrac{y+y^2}{1-2y}$　　C. $\dfrac{2y+1}{y+y^2}$　　D. $\dfrac{1-2y}{y+y^2}$　　E. $\dfrac{y+y^2}{1+2y}$

8. 考慮以下資料：14，5，7，7，8，8，9，10，11，m，n。若以上資料的平均值及中位數均為 9，則下列何者正確？

I. $m \geq 9$　　II. $n \leq 11$　　III. $m + n = 20$

A. 只有 I 及 II　　B. 只有 I 及 III　　C. 只有 II 及 III
D. I, II 及 III　　E. 以上皆非

9. 一組學生有 12 位女生和 3 位男生，從中隨機選出一名學生，其後再從餘下的學生中選出另一人。則選出男女生各一人的概率為

A. $\frac{2}{15}$　　B. $\frac{5}{12}$　　C. $\frac{2}{35}$　　D. $\frac{6}{35}$　　E. $\frac{12}{35}$

10. 某一無窮等比級數之和為 3，其各項的平方數之和為 45，則此級數首項為

A. 1　　B. 3　　C. 5　　D. $-\frac{2}{3}$　　E. -6

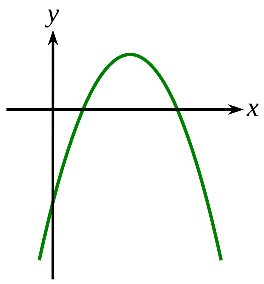
11. 橢圓 $9x^2+25y^2=225$ 的右焦點為

A. $(-3, 0)$　　B. $(3, 0)$　　C. $(0, -4)$　　D. $(0, 4)$　　E. $(4, 0)$

12. 用 1 到 9 這九個數字，組成沒有重複數字的三位數，其中奇數的個數為

A. 504　　B. 280　　C. 224　　D. 729　　E. 720

13. 函數 $y=a(x+b)^2+2$ 的圖形如右圖所示，其中 a 和 b 為常數。下列何者正確？



A. $a > 0$ 及 $b > 0$　　B. $a > 0$ 及 $b < 0$
C. $a < 0$ 及 $b > 0$　　D. $a < 0$ 及 $b < 0$
E. 以上皆非

14. 不等式 $|x-2|<2x$ 的解為

A. $x > \frac{2}{3}$　　B. $x<-2$ 或 $x > \frac{2}{3}$　　C. $-2<x<\frac{2}{3}$
D. $0<x<\frac{2}{3}$　　E. $x<-2$

15. 某個三角形的三邊之中點為 $A(3, 4)$, $B(2, 0)$ 和 $C(4, 2)$。下列哪一點是該三角形的一個頂點？

A. $(1, 2)$　　B. $(1, 3)$　　C. $(3, 1)$　　D. $(3, 2)$　　E. $(3.5, 3)$

第二部分 解答題。

1. 已知 $\sin\alpha=\frac{4}{5}$，$\alpha\in\left(\frac{\pi}{2},\pi\right)$。

(a) 求 $\sin\left(\alpha+\frac{\pi}{4}\right)$。 (4 分)

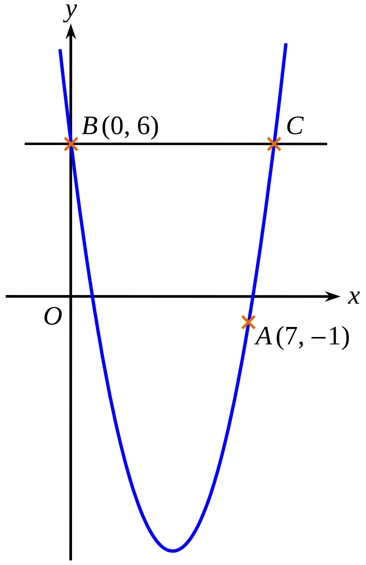
(b) 求 $\sin 2\alpha+\cos 2\alpha$。 (4 分)

2. 已知 $f(x)$ 為兩部分之和，一部分隨 x 正變，而另一部分隨 x^2 正變。假定 $f(2)=8$ 及 $f(6)=0$。

(a) 求 $f(x)$。 (4 分)

(b) 解方程 $\log_{\frac{1}{2}}\sqrt{f(x)}=-\frac{3}{2}$。 (4 分)

3. 在下圖中，$y=x^2-px+q$ 的圖像通過點 $A(7,-1)$，且與 y 軸交於點 $B(0,6)$。點 C 是圖像上的另一點，使得 BC 平行於 x 軸。



(a) 求 p 及 q 的值。 (3 分)

(b) 求點 C 的座標。 (3 分)

(c) 求 ΔABC 的面積。 (2 分)

4. 等比數列 $\{a_n\}_{n\geq 1}$ 的公比 $q>1$，$a_3a_7=16$ 和 $a_4+a_6=10$。

(a) 求 $\{a_n\}_{n\geq 1}$ 的通項。 (4 分)

(b) 求 $\{a_n\}_{n\geq 1}$ 的前 n 項的乘積 $T_n=a_1a_2\cdots a_n$。 (4 分)

5. (a) 若等式 $\sum_{k=1}^{n}\frac{1}{4k^2-1}=\frac{an}{bn+1}$ 對所有正整數 n 皆成立，其中 a 和 b 為常數，求 a 和 b 的值。 (4 分)

(b) 用數學歸納法證明等式 $\sum_{k=1}^{n}\frac{1}{4k^2-1}=\frac{an}{bn+1}$ 對所有正整數 n 皆成立，其中 a 和 b 為 (a) 部分確定的常數。 (4 分)

第一部分 選擇題。

題目編號	最佳答案
1	A
2	B
3	D
4	C
5	C
6	B
7	E
8	D
9	E
10	C
11	E
12	B
13	D
14	A
15	A

(第二部分答案由下頁開始)

第二部分 解答題。

1. (a) 因為 $\alpha\in\left(\frac{\pi}{2},\pi\right)$，所以 $\cos\alpha<0$，從而有 $\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\left(\frac{4}{5}\right)^2}=-\sqrt{\frac{9}{25}}=-\frac{3}{5}$。

由公式 $\sin(A+B)=\sin A\cos B+\cos A\sin B$，得

$$\sin\left(\alpha+\frac{\pi}{4}\right)=\sin\alpha\cos\frac{\pi}{4}+\cos\alpha\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}\sin\alpha+\frac{\sqrt{2}}{2}\cos\alpha=\frac{\sqrt{2}}{2}\cdot\frac{4}{5}+\frac{\sqrt{2}}{2}\cdot\left(-\frac{3}{5}\right)=\frac{\sqrt{2}}{10}\text{。}$$

(b) 由公式 $\sin 2\alpha=2\sin\alpha\cos\alpha$，得 $\sin 2\alpha=2\cdot\frac{4}{5}\cdot\left(-\frac{3}{5}\right)=-\frac{24}{25}$。

由公式 $\cos 2\alpha=1-2\sin^2\alpha$，得 $\cos 2\alpha=1-2\cdot\left(\frac{4}{5}\right)^2=-\frac{7}{25}$。

從而有

$$\sin 2\alpha+\cos 2\alpha=-\frac{24}{25}-\frac{7}{25}=-\frac{31}{25}\text{。}$$

2. (a) 據題意，$f(x)=ax+bx^2$，其中 a、b 為常數。

由 $f(2)=2a+4b=8$ 及 $f(6)=6a+36b=0$，得 $a=6$ 及 $b=-1$。所以 $f(x)=6x-x^2$。

(b) 由 $\log_{\frac{1}{2}}\sqrt{f(x)}=-\frac{3}{2}$，得 $\sqrt{f(x)}=\left(\frac{1}{2}\right)^{-\frac{3}{2}}=2^{\frac{3}{2}}=\sqrt{8}$。

從而有 $f(x)=8$，即 $6x-x^2=8$。解二次方程 $x^2-6x+8=0$ 得 $x=4$ 或 2。

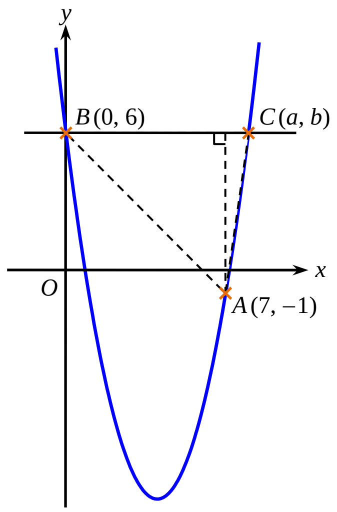
3. (a) 由於點 $A(7,-1)$ 與點 $B\,(0, 6)$ 在 $y=x^2-px+q$ 的圖像上，我們有 $7^2-7p+q=-1$ 及 $q=6$，從而有 $p=8$ 及 $q=6$。

(b) 設 C 的座標為 (a,b)。由於 BC 平行於 x 軸，$b=6$。因為點 C 在 $y=x^2-8x+6$ 的圖像上，所以 $a^2-8a+6=6$，即 $a^2-8a=0$，從而得 $a=8$ (由題意知，應捨棄 $a=0$)。

所以 C 的坐標為 $(8, 6)$。

(c) 點 $A(7,-1)$ 到 BC 的距離 $=6-(-1)=7$，為 ΔABC 中 BC 邊上的高的長度。由此及 $|BC|=8$ 得

$$\Delta ABC\text{ 的面積}=\frac{1}{2}\times 8\times 7=28\text{。}$$



4. (a) 由於 $4+6=3+7$，我們有 $a_4a_6=a_3a_7=16$。由此及 $a_4+a_6=10$，得知 a_4、a_6 為 $x^2-10x+16=0$ 的兩個根，即 $a_4=2$，$a_6=8$ (因為 $q>1$)，所以 $q^2=a_6/a_4=4$，即 $q=2$。

於是，$a_1=a_4/q^3=2/8=1/4$。因此通項公式為

$$a_n=a_1q^{n-1}=\frac{1}{4}\cdot 2^{n-1}=2^{n-3}\text{。}$$

(b) 由 (a) 的結果及等差級數公式得

$$T_n=a_1a_2\cdots a_n=2^{-2}\cdot 2^{-1}\cdot\cdots\cdot 2^{n-3}=2^{-2+(-1)+\cdots+(n-3)}=2^{\frac{n(-2+n-3)}{2}}=2^{\frac{n(n-5)}{2}}\text{。}$$

5. (a) 當 $n=1$ 時，有 $\frac{1}{4-1}=\frac{a}{b+1}$，即 $b+1=3a$。

當 $n=2$ 時，有 $\frac{1}{3}+\frac{1}{16-1}=\frac{2a}{2b+1}$，即 $2b+1=5a$。

解聯立方程組 $\begin{cases} b+1=3a \\ 2b+1=5a \end{cases}$，得 $a=1$ 及 $b=2$。

(b) 以下利用數學歸納法來證明 $\sum_{k=1}^{n}\frac{1}{4k^2-1}=\frac{n}{2n+1}$ 對所有正整數 n 皆成立。

I) 當 $n=1$ 時，左邊 $=\frac{1}{4-1}=\frac{1}{3}$，右邊 $=\frac{1}{2+1}=\frac{1}{3}$。

左邊 $=$ 右邊。命題成立。

II) 假設當 $n=\ell$ 時 ($\ell\in\mathbb{Z}^+$)，命題成立，即 $\sum_{k=1}^{\ell}\frac{1}{4k^2-1}=\frac{\ell}{2\ell+1}$。

當 $n=\ell+1$ 時，由歸納法假設得

$$\begin{aligned}\sum_{k=1}^{\ell+1}\frac{1}{4k^2-1}&=\sum_{k=1}^{\ell}\frac{1}{4k^2-1}+\frac{1}{4(\ell+1)^2-1}\\&=\frac{\ell}{2\ell+1}+\frac{1}{4\ell^2+8\ell+3}\\&=\frac{\ell}{2\ell+1}+\frac{1}{(2\ell+1)(2\ell+3)}\\&=\frac{\ell(2\ell+3)+1}{(2\ell+1)(2\ell+3)}\\&=\frac{(\ell+1)(2\ell+1)}{(2\ell+1)(2\ell+3)}\\&=\frac{\ell+1}{2\ell+3}\\&=\frac{\ell+1}{2(\ell+1)+1}\text{ 。}\end{aligned}$$

換句話說，當 $n=\ell+1$ 時，命題也成立。

綜合 I)、II) 和數學歸納法原理，可知 $\sum_{k=1}^{n}\frac{1}{4k^2-1}=\frac{n}{2n+1}$ 對所有正整數 n 皆成立。

<u>Part I</u> Multiple choice questions. Choose the *best answer* for each question.

1. If $X=\{6^n-5n-1 \mid n\in\mathbb{Z}^+\}$ and $Y=\{25n-25 \mid n\in\mathbb{Z}^+\}$, then

   A. $X\subset Y$ B. $Y\subset X$ C. $X=Y$ D. $X\cap Y=\varnothing$ E. none of the above

2. If $2x^3+x^2-29x+40$ is divided by $2x-5$, then the remainder is

   A. -5 B. 5 C. -20 D. 20 E. 30

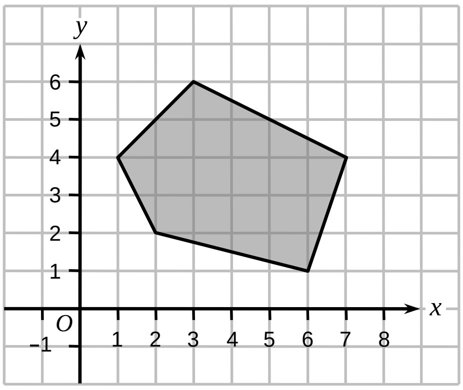
3. If the difference of the roots of the equation $3x^2-4x+k=0$ is $\frac{5}{3}$, then $k=$

   A. $\frac{2}{3}$ B. 3 C. $-\frac{1}{6}$ D. $-\frac{3}{4}$ E. none of the above

4. $\dfrac{a^3b^{-2}c^2}{\left(2a^{-1}b^2c\right)^3}=$

   A. $\dfrac{1}{8b^4c}$ B. $\dfrac{a}{8b^4c}$ C. $\dfrac{a^6}{8b^8c}$ D. $\dfrac{a^4}{8b^4c}$ E. $\dfrac{a^6b^3}{8c}$

5. Find the largest possible value of $M=3x+2y-6$ if (x, y) is any point lying within the shaded region (including the boundary) as shown in the figure below.



   A. 4 B. 14 C. 23 D. 32 E. none of the above

6. If a, $b>1$, what is the maximum value of $\log_a\left(\frac{a}{b}\right)+\log_b\left(\frac{b}{a}\right)$?

   A. -2 B. 0 C. 2 D. 3 E. 4

7. If $\dfrac{x}{y-x}=\dfrac{1+y}{y}$, then $x=$

   A. $y+y^2$ B. $\dfrac{y+y^2}{1-2y}$ C. $\dfrac{2y+1}{y+y^2}$ D. $\dfrac{1-2y}{y+y^2}$ E. $\dfrac{y+y^2}{1+2y}$

8. Consider the numbers: 14, 5, 7, 7, 8, 8, 9, 10, 11, m, n. If the mean and median are both 9, which of the following is true?

I. $m \geq 9$ II. $n \leq 11$ III. $m + n = 20$

A. I and II only B. I and III only C. II and III only
D. I, II and III E. none of the above

9. There are 12 females and 3 males in a group of students. One student is chosen at random from the group. Another one is chosen at random from the remaining students. Calculate the probability that two students of different genders are chosen.

A. $\frac{2}{15}$ B. $\frac{5}{12}$ C. $\frac{2}{35}$ D. $\frac{6}{35}$ E. $\frac{12}{35}$

10. The sum to infinity of a geometric series is 3, and the sum of the squares of each of its terms is 45. Its first term is

A. 1 B. 3 C. 5 D. $-\frac{2}{3}$ E. -6

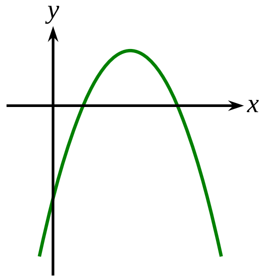
11. Which of the following is the right focus of the ellipse $9x^2 + 25y^2 = 225$?

A. $(-3, 0)$ B. $(3, 0)$ C. $(0, -4)$ D. $(0, 4)$ E. $(4, 0)$

12. 3-digit numbers are constructed by taking three different digits from 1, 2, 3, …, 9. How many of these constructed numbers are odd?

A. 504 B. 280 C. 224 D. 729 E. 720

13. The right figure shows the graph of $y = a(x+b)^2 + 2$, where a and b are constants. Which of the following is true?



A. $a > 0$ and $b > 0$ B. $a > 0$ and $b < 0$
C. $a < 0$ and $b > 0$ D. $a < 0$ and $b < 0$
E. none of the above

14. The solution of the inequality $|x - 2| < 2x$ is

A. $x > \frac{2}{3}$ B. $x < -2$ or $x > \frac{2}{3}$ C. $-2 < x < \frac{2}{3}$
D. $0 < x < \frac{2}{3}$ E. $x < -2$

15. The midpoints of the sides of a triangle are $A(3, 4)$, $B(2, 0)$ and $C(4, 2)$. Which of the following points is a vertex of the triangle?

A. $(1, 2)$ B. $(1, 3)$ C. $(3, 1)$ D. $(3, 2)$ E. $(3.5, 3)$

<u>Part II</u> Problem-solving questions.

1. Suppose that $\sin\alpha = \frac{4}{5}$, $\alpha \in \left(\frac{\pi}{2}, \pi\right)$.

   (a) Find $\sin\left(\alpha + \frac{\pi}{4}\right)$. (4 marks)

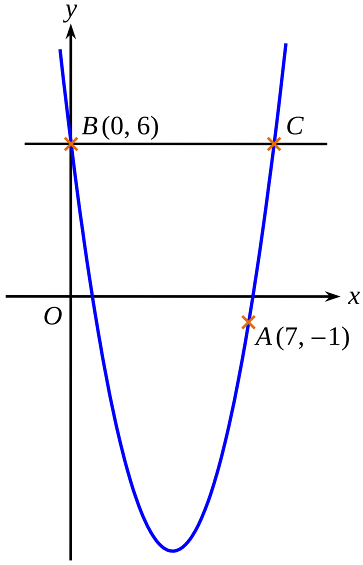
   (b) Find $\sin 2\alpha + \cos 2\alpha$. (4 marks)

2. Suppose $f(x)$ is a sum of two parts, the first part varies directly with x, and the other part varies directly with x^2. Assume that $f(2)=8$ and $f(6)=0$.

   (a) Find $f(x)$. (4 marks)

   (b) Solve the equation $\log_{\frac{1}{2}}\sqrt{f(x)} = -\frac{3}{2}$. (4 marks)

3. In the below figure, the graph of $y=x^2-px+q$ passes through $A(7,-1)$ and intersects the y-axis at $B(0, 6)$. C is another point on the graph such that BC is parallel to the x-axis.



   (a) Find the values of p and q. (3 marks)

   (b) Find the coordinates of C. (3 marks)

   (c) Find the area of ΔABC. (2 marks)

4. In a geometric sequence $\{a_n\}_{n\geq 1}$, the common ratio $q>1$, $a_3a_7=16$ and $a_4+a_6=10$.

   (a) Find the general term of $\{a_n\}_{n\geq 1}$. (4 marks)

   (b) Find the product T_n of the first n terms of $\{a_n\}_{n\geq 1}$, i.e., $T_n=a_1a_2\cdots a_n$. (4 marks)

5. (a) If the equality $\sum_{k=1}^{n}\frac{1}{4k^2-1} = \frac{an}{bn+1}$ holds for all positive integers n, where a and b are constants, find the values of a and b. (4 marks)

   (b) Prove by mathematical induction that for all positive integers n, the equality $\sum_{k=1}^{n}\frac{1}{4k^2-1} = \frac{an}{bn+1}$ holds, where a and b are the constants found in (a). (4 marks)

Part I **Multiple choice questions.**

Question Number	Best Answer
1	A
2	B
3	D
4	C
5	C
6	B
7	E
8	D
9	E
10	C
11	E
12	B
13	D
14	A
15	A

(Answers for Part II start from next page)

<u>Part II</u> Problem-solving questions.

1. (a) Since $\alpha \in \left(\frac{\pi}{2}, \pi\right)$, $\cos\alpha < 0$. Hence $\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\left(\frac{4}{5}\right)^2} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$.

From the formula $\sin(A+B) = \sin A\cos B + \cos A\sin B$, we have

$$\sin\left(\alpha+\frac{\pi}{4}\right) = \sin\alpha\cos\frac{\pi}{4} + \cos\alpha\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}\sin\alpha + \frac{\sqrt{2}}{2}\cos\alpha = \frac{\sqrt{2}}{2}\cdot\frac{4}{5} + \frac{\sqrt{2}}{2}\cdot\left(-\frac{3}{5}\right) = \frac{\sqrt{2}}{10}.$$

(b) From the formula $\sin 2\alpha = 2\sin\alpha\cos\alpha$, we have $\sin 2\alpha = 2\cdot\frac{4}{5}\cdot\left(-\frac{3}{5}\right) = -\frac{24}{25}$.

From the formula $\cos 2\alpha = 1-2\sin^2\alpha$, we have $\cos 2\alpha = 1-2\cdot\left(\frac{4}{5}\right)^2 = -\frac{7}{25}$.

Eventually, we have

$$\sin 2\alpha + \cos 2\alpha = -\frac{24}{25} - \frac{7}{25} = -\frac{31}{25}.$$

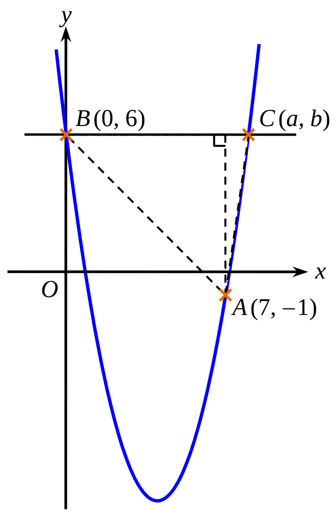
2. (a) The given means that $f(x) = ax + bx^2$ for some constants a and b.

From $f(2) = 2a + 4b = 8$ and $f(6) = 6a + 36b = 0$, we get $a = 6$ and $b = -1$. Therefore, $f(x) = 6x - x^2$.

(b) From $\log_{\frac{1}{2}}\sqrt{f(x)} = -\frac{3}{2}$, we get $\sqrt{f(x)} = \left(\frac{1}{2}\right)^{-\frac{3}{2}} = 2^{\frac{3}{2}} = \sqrt{8}$.

It follows that $f(x) = 8$, i.e., $6x - x^2 = 8$. Solving the quadratic equation $x^2 - 6x + 8 = 0$ yields $x = 4$ or 2.

3. (a) Since the point $A(7, -1)$ and the point $B(0, 6)$ lie on the graph of $y = x^2 - px + q$, we get $7^2 - 7p + q = -1$ and $q = 6$, and so $p = 8$ and $q = 6$.



(b) Let the coordinates of C be (a, b). Since BC is parallel to x-axis, we have $b = 6$. Also, since C lies on the graph of $y = x^2 - 8x + 6$, $a^2 - 8a + 6 = 6$, i.e., $a^2 - 8a = 0$, and so $a = 8$ ($a = 0$ is discarded according to the given).

Therefore, the coordinates of C are (8, 6).

(c) The distance from $A(7, -1)$ to $BC = 6 - (-1) = 7$, which is the height of ΔABC with BC as the base. This, together with $|BC| = 8$, gives

$$\text{area of } \Delta ABC = \frac{1}{2}\times 8\times 7 = 28.$$

4. (a) Since $4+6 = 3+7$, we have $a_4a_6 = a_3a_7 = 16$. This, together with $a_4 + a_6 = 10$, implies that a_4 and a_6 are the two roots of $x^2 - 10x + 16 = 0$, i.e., $a_4 = 2$ and $a_6 = 8$ (because $q > 1$), therefore, $q^2 = a_6/a_4 = 4$, i.e., $q =$ 2.

Hence, $a_1 = a_4/q^3 = 2/8 = 1/4$. The general term is thus given by

$$a_n = a_1q^{n-1} = \frac{1}{4}\cdot 2^{n-1} = 2^{n-3}.$$

(b) From the result of (a) and the formula for arithmetic series, we get

$$T_n = a_1a_2\cdots a_n = 2^{-2}\cdot 2^{-1}\cdot\cdots\cdot 2^{n-3} = 2^{-2+(-1)+\cdots+(n-3)} = 2^{\frac{n(-2+n-3)}{2}} = 2^{\frac{n(n-5)}{2}}.$$

5. (a) When $n=1$, $\frac{1}{4-1}=\frac{a}{b+1}$, i.e., $b+1=3a$.

When $n=2$, $\frac{1}{3}+\frac{1}{16-1}=\frac{2a}{2b+1}$, i.e., $2b+1=5a$.

Solving the system of equations $\begin{cases} b+1=3a \\ 2b+1=5a \end{cases}$, we obtain $a=1$ and $b=2$.

(b) In what follows, we want to prove by mathematical induction that $\sum_{k=1}^{n}\frac{1}{4k^2-1}=\frac{n}{2n+1}$ holds for any positive integer n.

I) When $n=1$, LHS $=\frac{1}{4-1}=\frac{1}{3}$, and RHS $=\frac{1}{2+1}=\frac{1}{3}$.

LHS $=$ RHS. The statement is true.

II) Assume that when $n=\ell$ ($\ell\in\mathbb{Z}^+$), the statement is true, i.e., $\sum_{k=1}^{\ell}\frac{1}{4k^2-1}=\frac{\ell}{2\ell+1}$.

When $n=\ell+1$, it follows from the induction assumption that

$$\begin{aligned}\sum_{k=1}^{\ell+1}\frac{1}{4k^2-1}&=\sum_{k=1}^{\ell}\frac{1}{4k^2-1}+\frac{1}{4(\ell+1)^2-1}\\&=\frac{\ell}{2\ell+1}+\frac{1}{4\ell^2+8\ell+3}\\&=\frac{\ell}{2\ell+1}+\frac{1}{(2\ell+1)(2\ell+3)}\\&=\frac{\ell(2\ell+3)+1}{(2\ell+1)(2\ell+3)}\\&=\frac{(\ell+1)(2\ell+1)}{(2\ell+1)(2\ell+3)}\\&=\frac{\ell+1}{2\ell+3}\\&=\frac{\ell+1}{2(\ell+1)+1}.\end{aligned}$$

In other words, the statement is also true when $n=\ell+1$.

According to I), II), and the Principle of Mathematical Induction, $\sum_{k=1}^{n}\frac{1}{4k^2-1}=\frac{n}{2n+1}$ is true for any positive integer n.